Quantitative Easing and Balance Sheet Policy

Long-Term Bonds

Consider a long-term bond issued at time t that pays coupons χ^s at time $t+s$, for $s > 0$, with $0 < \chi < 1$.

Define the total dollar payment to households at time $t+1$ from holdings of long-term bonds as

$$\widetilde{B}_{Lt} = \sum_{j=0}^{\infty} \chi^j NB_{L,t-j}, \tag{1}$$

where NB_{Lt} denotes purchases of long-term bonds issued at time t.

(a) Show that

$$NB_{Lt} = \widetilde{B}_{Lt} - \chi \widetilde{B}_{L,t-1}. \tag{2}$$

(b) Explain why this specification is convenient relative to modelling a full term structure with many maturities.

Household Budget Constraint

Suppose the household budget constraint is

$$P_t C_t + B_t^h + V_t NB_{Lt}^h = W_t N_t + (1+i_{t-1}) B_{t-1}^h + \widetilde{B}_{L,t-1}^h - T_t, \tag{3}$$

where V_t is the price of the long-term bond and $B^h_{\cdot}$ denotes bond holding by households.

(a) Using the identity from the previous part, show that the budget constraint can be rewritten as

$$P_t C_t + B_t^h + V_t \widetilde{B}_{Lt}^h = W_t N_t + (1+i_{t-1}) B_{t-1}^h + (1+\chi V_t) \widetilde{B}_{L,t-1}^h - T_t. \tag{4}$$

(b) Define

$$B_{Lt} \equiv V_t \widetilde{B}_{Lt}, \qquad 1 + i_{Lt} \equiv \frac{1+\chi V_t}{V_{t-1}}, \tag{5}$$

and show that the household budget constraint becomes

$$P_t C_t + B_t^h + B_{Lt}^h = W_t N_t + (1+i_{t-1}) B_{t-1}^h + (1+i_{Lt}) B_{L,t-1}^h - T_t. \tag{6}$$

(c) Explain the economic meaning of i_{Lt}.

Preferred Habitat

Assume households have preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{\nu}_b}{2} \left(\frac{B_t^h}{B_{Lt}^h} - \delta \right)^2 - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{7}$$

where $\tilde{\nu}_b > 0$ and $\delta \equiv B^h / B_L^h$ is the target ratio of short-term to long-term bonds.

(a) Explain why this preference specification captures a form of market segmentation or "preferred habitat".

(b) Explain what happens when the actual ratio B_t^h / B_{Lt}^h differs from the target δ.

Euler Equations

(b) Starting from the household problem, derive the Euler equations:

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} \frac{1+i_t}{\Pi_{t+1}} \right] - \frac{\tilde{\nu}_b}{B_{Lt}^h/P_t} \left(\frac{B_t^h/P_t}{B_{Lt}^h/P_t} - \delta \right), \tag{8}$$

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} \frac{1+i_{L,t+1}}{\Pi_{t+1}} \right] + \frac{\tilde{\nu}_b}{B_{Lt}^h/P_t} \left(\frac{B_t^h/P_t}{B_{Lt}^h/P_t} - \delta \right) \frac{B_t^h/P_t}{B_{Lt}^h/P_t}. \tag{9}$$

(b) Explain why the portfolio wedge enters the short-bond Euler equation with one sign and the long-bond Euler equation with the opposite sign.

(c) Show that, up to a log-linear approximation, these two Euler equations become

$$-\sigma c_t = -\sigma E_t c_{t+1} + i_t - E_t \pi_{t+1} - \nu_b (b_t^h - b_{Lt}^h), \tag{10}$$

$$-\sigma c_t = -\sigma E_t c_{t+1} + E_t i_{L,t+1} - E_t \pi_{t+1} + \nu_b \delta (b_t^h - b_{Lt}^h), \tag{11}$$

where

$$\nu_b \equiv \tilde{\nu}_b \delta C^\sigma / (B_L^h/P). \tag{12}$$

Term Premium

(a) Subtract the short-bond Euler equation from the long-bond Euler equation and show that

$$E_t i_{L,t+1} - i_t = \nu_b (1+\delta)(b_{Lt}^h - b_t^h). \tag{13}$$

(b) Explain why this expression can be interpreted as a term-premium equation.

(c) Using the sign of the right-hand side, explain why a QE purchase of long-term bonds by the central bank lowers the term premium.

Modified Aggregate Demand

(a) Replace the portfolio wedge in the short-bond Euler equation using the term-premium relation and show that

$$c_t = E_t c_{t+1} - \sigma^{-1} \left(\frac{\delta}{1+\delta} i_t + \frac{1}{1+\delta} E_t i_{L,t+1} - E_t \pi_{t+1} \right). \tag{14}$$

(b) Explain why both short-term and long-term interest rates now matter for aggregate demand.

(c) Explain why this is especially relevant at the ZLB.

Government and Central Bank

Consider the government budget constraint

$$B_t + B_{Lt} = (1+i_{t-1}) B_{t-1} + (1+i_{Lt}) B_{L,t-1} - Z_t - T_t, \tag{15}$$

where, after denoting with $B_\cdot^{cb}$ bonds hold by the monetary authority, transfers from the central bank are

$$Z_t = (1+i_{t-1}) B_{t-1}^{cb} + (1+i_{Lt}) B_{L,t-1}^{cb} - B_t^{cb} - B_{Lt}^{cb}. \tag{16}$$

(a) Write down the household budget constraint with short-term and long-term bonds.

(b) Substitute out taxes using the government budget constraint and the central-bank transfer equation.

(c) Using bond-market clearing,

$$B_t = B_t^h + B_t^{cb}, \qquad B_{Lt} = B_{Lt}^h + B_{Lt}^{cb}, \tag{17}$$

show that the overall level of government debt does not matter for equilibrium.

Fiscal Rules and the Asset Purchase Rule

Assume the fiscal rules

$$\frac{B_t}{P_t} = \delta b, \qquad \frac{B_{Lt}}{P_t} = b, \tag{18}$$

and the asset purchase rule

$$\frac{B_t^{cb}}{P_t} + \frac{B_{Lt}^{cb}}{P_t} = B. \tag{19}$$

(a) Explain the meaning of these assumptions.

(b) Show that, up to a first-order log-linear approximation, market clearing implies

$$b^h b_t^h + b^{cb} b_t^{cb} = 0, \qquad b_L^h b_{Lt}^h + b_L^{cb} b_{Lt}^{cb} = 0. \tag{20}$$

(c) Show that the asset purchase rule implies

$$b^{cb} b_t^{cb} + b_L^{cb} b_{Lt}^{cb} = 0. \tag{21}$$

(d) Combine these relations to show that

$$b_t^h = -\frac{b_L^h}{b^h} b_{Lt}^h. \tag{22}$$

(e) Using the fiscal rules, market-clearing conditions, and the asset purchase rule, show that the term premium $E_t i_{L,t+1} - i_t = \nu_b(1+\delta)(b_{Lt}^h - b_t^h)$ can be written in terms of central bank holdings of long-term bonds only:

$$E_t i_{L,t+1} - i_t = -\frac{\nu_b(1+\delta)^2}{\delta} \frac{b_L^{cb}}{b_L^h} b_{Lt}^{cb}. \tag{23}$$

(f) Discuss the sign of this expression. What happens to the term premium when the central bank increases its holdings of long-term bonds?

(g) Substitute the previous result into aggregate demand and show that

$$x_t = E_t x_{t+1} - \sigma^{-1} \left[i_t - \frac{\nu_b(1+\delta)}{\delta} \frac{b_L^{cb}}{b_L^h} b_{Lt}^{cb} - E_t \pi_{t+1} - r_t^* \right]. \tag{24}$$

(h) Explain why an increase in central bank holdings of long-term bonds stimulates aggregate demand.

(i) Show that QE and conventional interest-rate policy are substitutes in this model.

(j) Find the change in central bank holdings of long-term debt, relative to steady state, that has the same effect on aggregate demand as a 1 percentage point cut in the nominal interest rate.

(k) Explain why the effectiveness of QE depends on ν_b, δ, and the ratio b_L^h / b_L^{cb}.

(l) Explain why, if steady-state central bank holdings of long-term debt are small, the required QE intervention may have to be large.