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Application for the MPhil in Economics

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Content:

Humanizing Macroeconomics: Beyond Rationality (2000 words, 37 math inlines)

MATLAB codes are provided in the Appendix

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Humanizing Macroeconomics: Beyond Rationality

Abstract*

Macroeconomic models have long incorporated the hypothesis of rational expectations. This premise is pivotal in ensuring consistency and facilitating equilibrium; however, it raises numerous inquiries. As Douglass C. North puts it, we must ascertain ”What Do We Mean by Rationality?” Is the implication that agents possess omniscience and consistently maximize utility? Research by psychologists such as Kahneman indicates otherwise. Additionally, individuals are susceptible to a ”contagion-effect,” as brilliantly introduced by Kirman (1993), where they are inclined to conform to the most widely recognized opinion. In this context, I will present a simple DSGE model with rational expectations and elucidate three of its limitations. Subsequently, I will introduce and simulate a Behavioural Model, specifically a variation of De Grauwe (2008), to address these deficiencies.

1 The DSGE Framework

Following the rational expectations revolution, wherein Lucas (1976) expanded on the concept introduced by Muth (1961), the Real Business Cycle (RBC) model, distinguished by its micro-foundations and rational expectations, emerged. This model's equations are derived from the optimizing behaviour of economic agents. Furthermore, both consumers and firms possess rational expectations, implying that they have perfect knowledge of the model's structure and the distributions of economic shocks. Given that all agents are rational and share the same information set, the model focuses on a representative agent. The initial model, however, assumed perfect price flexibility, the absence of monetary factors, and solely a Total Factor Productivity (TFP) shock. This led to the development of Dynamic Stochastic General Equilibrium (DSGE) models, which incorporate sticky prices, thus legitimizing the role of a central bank, and consider multiple shocks to disturb the economy. These models have now achieved widespread application in both policy-making and academia (see Smets and Wouters, 2003, 2005; Christoffel et al., 2008; Smets et al.,

*included in the word count

2010). A simplified model can be articulated using three fundamental equations, delineating aggregate demand, the supply curve, and the Central Bank's policy rule. The aggregate demand is derived by maximizing the preferences of the representative household, incorporating habit formation as discussed in Abel et al. (1990) to ensure that the impulse response function (IRF) of output conforms to the empirically observed hump-shaped patterns. The household thus maximises the following:

$$E_{t-1}\sum_{i=0}^{\infty}\beta^{i-t}\left(u\left(X_{t+i}-hX_{t+i-1}\right)-v\left(N_{t+i}\right)\right)$$

Where E_{t-1} is the expectation operator, β is the discount factor, X_t and N_t respectively represent output consumed and hours worked at time t, while $h>0$ allows for habit formation. Furthermore, households are subject to a series of budget constraints of the form:

$$P_tX_t+Q_tB_t=B_{t-1}+W_tN_t$$

where P_t is the price of goods, W_t the nominal hourly wage, B_t the quantity of riskless bonds acquired at time t and maturing in $t+1$. These bonds offer a unit payoff at maturity and are priced at Q_t. By resolving the optimization problem and log-linearizing, the demand curve is obtained:

$$x_t=a_1E_tx_{t+1}+(1-a_1)x_{t-1}+a_2\left(r_t-E_t\pi_{t+1}\right)+\varepsilon_t \tag{1}$$

The supply equation, known as the New Keynesian Philips curve, embodies both forward-looking and backward-looking inflation components. Within classical literature, Calvo pricing is extensively employed to model price stickiness: within this framework, each firm has the probability $1-\theta$ to reset its prices at time t, and must continue with unchanged prices at probability θ. Here, however, to more accurately reflect the observed inflation inertia, firms unable to reoptimize their prices align them with past inflation, as elucidated by Christiano et al. (2001). Firms then select P_t^* to maximize the following objective function:

$$E_{t-1}\sum_{i=0}^{\infty}(\beta\theta)^i\lambda_{t+i}\left(P_t^*\Pi_{ti}-MC_{t+i}\right)X_{j,t+i}$$

$$\text{where } \Pi_{ti}=\begin{cases}\pi_t\times\pi_{t+1}\times\cdots\times\pi_{t+i-1} & \text{for } i\geq 1\\ 1 & i=0\end{cases}$$

subject to:

$$X_{j,t+i} = \left(\frac{P_t^*}{P_{t+i}}\right)^{-\varphi} X_{t+i}$$

where λ_t is the marginal utility out of income for the households, φ is the elasticity of substitution between goods, MC_t the marginal cost, and $X_{j,t+i}$ is output in period $t+i$ for a firm that resets its price at time t. The solution to this problem yields:

$$\pi_t = b_1 E_t \pi_{t+1} + (1-b_1)\,\pi_{t-1} + b_2 x_t + \eta_t \tag{2}$$

Finally, the model is closed using the Taylor Rule, that represent the reaction of the Central Bank to changes in output gap x_t and inflation π_t.

$$r_t = c_1\,(\pi_t - \pi_t^*) + c_2 x_t + c_3 r_{t-1} + u_t \tag{3}$$

In this context, π_t^* represents the inflation target set by the Central Bank, and as dictated by the Taylor Principle, $c_1 > 1$. Furthermore, $c_2 > 0$ and $c_3 > 0$ is the smoothing parameter. The terms $\varepsilon_t, \eta_t, u_t$ in equations 1, 2, 3 are construed as shocks, which are characterized by disturbance terms following a normal distribution.

While the model is capable of reproducing stylized facts and can emulate the movements of the business cycle, it nevertheless exhibits certain limitations. Firstly, it fails to replicate the higher moments of the business cycle. Empirical observations of the output gap reject the normality hypothesis for U.S. data (De Grauwe, 2012), showing excess kurtosis. A similar conclusion applies to EU data, as shown in figure 1, though at a less significant level. Conversely, DSGE models tend to generate output gap movements that follow a normal distribution. This issue is critical, as disregarding the presence of fat tails diminishes the model's capability to represent and predict extreme events. Taleb (2007) addresses this problem through a philosophical lens. Secondly, within DSGE models, economic booms and busts originate solely from exogenous shocks. For instance, it is common in literature to introduce various types of shocks to model crises or recessions. Moreover, the transmission of these shocks is predictable and remains unaffected by the prevailing state of the economy. In simple terms, the same shock consistently yields identical outcomes, irrespective of market sentiment. Thirdly, these models assume the existence of an omniscient agent

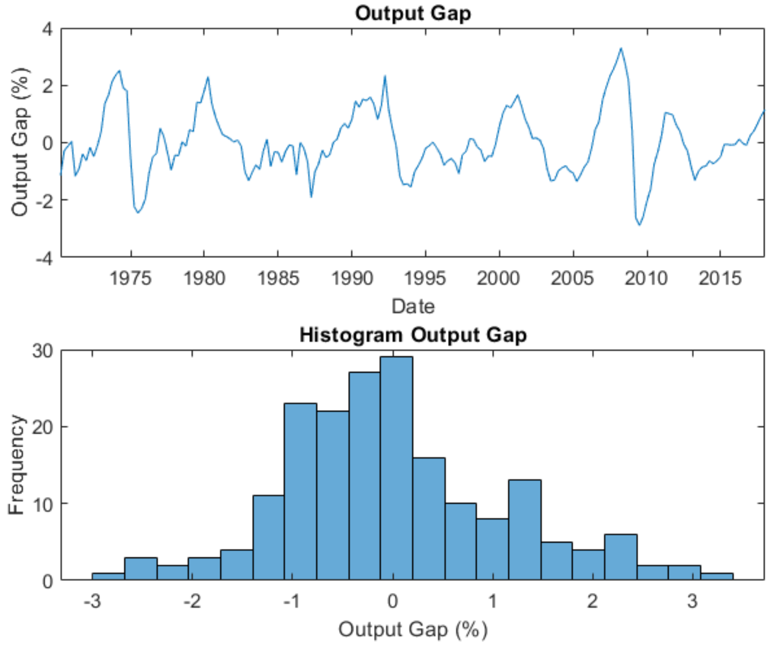

Figure 1: Frequency distribution of EU Output Gap, 1970Q1:2017Q4. Excess kurtosis=3.2, Jarque-Bera=4.8 (p-value=0.07)

Source: produced by the author. The data set is the one used by Fagan et al. (2001), discontinued in 2018. Output is detrended using an HP filter ($\lambda = 1600$) to estimate the output gap.

endowed with rational expectations, who operates optimally and with an infinite temporal horizon for optimization. This assumption has been repudiated by substantial experimental evidence: agents are frequently influenced by their context or emotional state, as documented by Kahneman and Thaler (2006); they make decisions that do not maximize utility, as shown by Kahneman and Tversky (1979); and their choices are often time-inconsistent, as demonstrated by Thaler (1981). Consequently, there is a pressing need to augment this framework to adequately address these inconsistencies.

2 Adaptive Learning Model

2.1 The model

De Grauwe (2008) revisits the concept of ”animal spirits” as originally introduced by Keynes. These phenomena are characterized by fluctuating levels of optimism and pessimism that have self-fulfilling properties, arising from the fact that individuals possess only partial knowledge of the information set, thereby exhibiting heterogeneous behaviours. Given their limited capacity to fully comprehend the complexities of the world, agents resort to simple heuristics, subsequently adopting those that yield superior outcomes. This ”trial and error” approach is known as ”adaptive learning”, for reasons that will become clear later.
The equations of this model are akin to those established for the DSGE baseline model:

$$x_t = a_1 \tilde{E}_t x_{t+1} + (1 - a_1) x_{t-1} + a_2 \left(r_t - \tilde{E}_t \pi_{t+1} \right) + \varepsilon_t \tag{4}$$

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 x_t + \eta_t \tag{5}$$

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 x_t + c_3 r_{t-1} + u_t \tag{6}$$

where the tilde over the expectations operator signifies that expectations for future output and inflation are not formed rationally. Pertaining to the former, it is postulated that certain agents maintain an optimistic outlook, while others hold a pessimistic view, and a third group extrapolates from the most recent observation. The first group consistently skews output predictions upwards, and the second consistently delivers negative forecasts; these are identified as the fundamentalists. The methodologies are delineated as follows:

$$\text{optimistic } (f_o)\text{: } \tilde{E}_t^{f_o} x_{t+1} = g_t \tag{7}$$

$$\text{pessimistic } (f_p)\text{: } \tilde{E}_t^{f_p} x_{t+1} = -g_t \tag{8}$$

$$\text{extrapolative: } \tilde{E}_t^{e} x_{t+1} = x_{t-1} \tag{9}$$

where $g_t = k + \delta var(x_t)$, $k > 0$, $\delta > 0$

thus the forecasts of output gap of fundamentalists tend to diverge more when the uncertainty about output increases. The collective market forecast for output is then derived as a weighted mean of these three clusters:

$$\tilde{E}_t x_{t+1} = \alpha_{f_o,t} \tilde{E}_t^{f_o} x_{t+1} + \alpha_{f_p,t} \tilde{E}_t^{f_p} x_{t+1} + \alpha_{e,t} x_{t-1} \tag{10}$$

$$\text{where } \alpha_{f_o,t} + \alpha_{f_p,t} + \alpha_{e,t} = 1$$

At each period's conclusion, agents assess the forecast accuracy of the various heuristics by calculating the mean squared forecasting errors (MSFEs) of the rules employed as follows:

$$U_{f_o,t} = -\sum_{k=1}^{\infty} \omega_k \left(x_{t-k} - \tilde{E}_{t-k-1}^{f_o} x_{t-k} \right)^2 \tag{11}$$

$$U_{f_p,t} = -\sum_{k=1}^{\infty} \omega_k \left(x_{t-k} - \tilde{E}_{t-k-1}^{f_p} x_{t-k} \right)^2 \tag{12}$$

$$U_{e,t} = -\sum_{k=1}^{\infty} \omega_k \left(x_{t-k} - \tilde{E}_{t-k-1}^{e} x_{t-k} \right)^2 \tag{13}$$

where ω_k are geometrically diminishing weights, reflecting agents' tendencies to place diminished emphasis on past errors, a behaviour commonly known as recency bias. Subsequently, the agents must evaluate these utilities: assuming pure rationality, they would opt for the rule delivering the highest utility. Nevertheless, empirical research, as thoroughly reviewed by Della Vigna (2007), indicates that individuals are influenced by external factors in their decision-making processes. This implies that the utilities of the three options comprise both deterministic and stochastic components. De Grauwe (2008) models the probability of selecting different rules following the approach of Brock and Hommes (1997): ,

$$\alpha_{f_o,t} = \frac{e^{\gamma U_{f_o,t}}}{e^{\gamma U_{f_o,t}} + e^{\gamma U_{f_p,t}} + e^{\gamma U_{e,t}}} \tag{14}$$

$$\alpha_{f_p,t} = \frac{e^{\gamma U_{f_p,t}}}{e^{\gamma U_{f_o,t}} + e^{\gamma U_{f_p,t}} + e^{\gamma U_{e,t}}} \tag{15}$$

$$\alpha_{e,t} = \frac{e^{\gamma U_{e,t}}}{e^{\gamma U_{f_o,t}} + e^{\gamma U_{f_p,t}} + e^{\gamma U_{e,t}}} \tag{16}$$

where the parameter γ indicates the degree of "willingness to learn from past performances": at $\gamma = 0$, probabilities are distributed uniformly, indicating indifference towards rule efficacy; at $\gamma \to \infty$, agents invariably prefer the rule with superior performance.

The agents employ an analogous set of rules to forecast inflation. The fundamentalists possess confidence that the central bank will effectively regulate inflation, and consequently, they utilize its target for their forecasts. Conversely, agents who perceive the central bank's announcements as lacking credibility resort to employing the extrapolative rule. Hence,

$$\tilde{E}_t^{fun}\pi_{t+1} = \pi_t^* \tag{17}$$

$$\tilde{E}_t^{ext}\pi_{t+1} = \pi_{t-1} \tag{18}$$

$$\text{where } \tilde{E}_t\pi_{t+1} = \beta_{fun,t}\pi_t^* + \beta_{ext,t}\pi_{t-1}$$

$$\text{and } \pi_t^* = 0$$

the performance and utility of these rules are evaluated by agents through a mechanism akin to that used for predicting output:

$$\beta_{fun,t} = \frac{e^{\gamma U_{fun,t}}}{e^{\gamma U_{fun,t}} + e^{\gamma U_{ext,t}}} \tag{19}$$

$$\beta_{ext,t} = \frac{e^{\gamma U_{ext,t}}}{e^{\gamma U_{fun,t}} + e^{\gamma U_{ext,t}}} \tag{20}$$

where $U_{fun,t}$ and $U_{ext,t}$ represent the forecast performances of the fundamentalist and extrapolative rules, respectively, as determined in eq.11 and 13.

2.2 Simulation of the model

The model is calibrated in accordance with the methodologies outlined in De Grauwe (2008), whereby temporal units are represented in months rather than quarters. Within the simulation, the economy is continuously subjected to demand and supply shocks, which are i.i.d. and exhibit no autocorrelation, possessing standard deviations of 0.5%. The findings of the exercise are illustrated in figures 2 and 3. Panel 1 of figure 2 illustrates the temporal pattern of output, characterized by cyclical fluctuations interspersed with periods of relatively low volatility, akin to those depicted in figure 1. The lower panel of the same figure elucidates the origins of these movements, attributed to the proportion of optimists and pessimists within the market, an embodiment of the concept of animal spirits. The model thus facilitates the "generation of endogenous waves of optimism and pessimism". These cycles are sustained by a self-fulfilling mechanism: as random shocks perturb the economic landscape, one of the three decision rules yields a superior payoff,

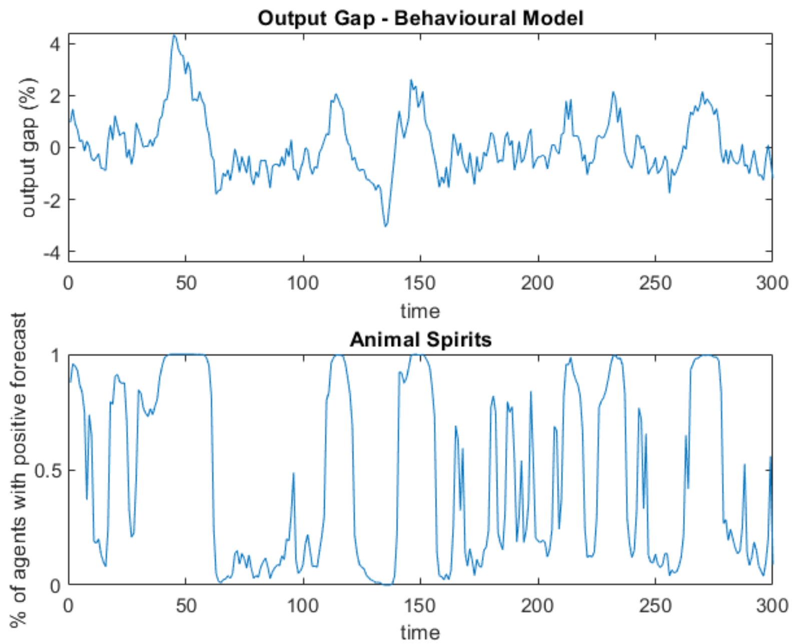

Figure 2: Output Gap in the Behavioural Model

Source: produced by the author, based on De Grauwe (2012)

hypothetically the pessimistic one. Consequently, this draws agents who previously adhered to the optimistic or extrapolative rule. This ”herding effect” augments the proportion of pessimistic agents, thereby diminishing aggregate demand and suppressing output, a phenomenon observable after periods 50 and 150 in the chart. When a subsequent shock renders the optimistic rule more attractive, it likewise attracts adherents, thereby initiating a trend reversal and leading to an economic boom.

Inflation exhibits distinctive behaviour. When the percentage of forecasters adhering to the central bank's inflation target (ECB followers in the charts) stabilizes around 50%, the inflation rate remains within 1% of the target. However, an increase in the proportion of extrapolators leads to greater fluctuations in inflation, thereby rendering the central bank's inflation targeting mechanism considerably fragile. This vulnerability is further intensified by its intrinsic unpredictability, as fluctuations may occur due to unforeseen shocks, which consequently render reliance on historical inflation data more favourable compared to confidence in the bank's target.

The model successfully addresses the three limitations inherent in its rational expectations ana-

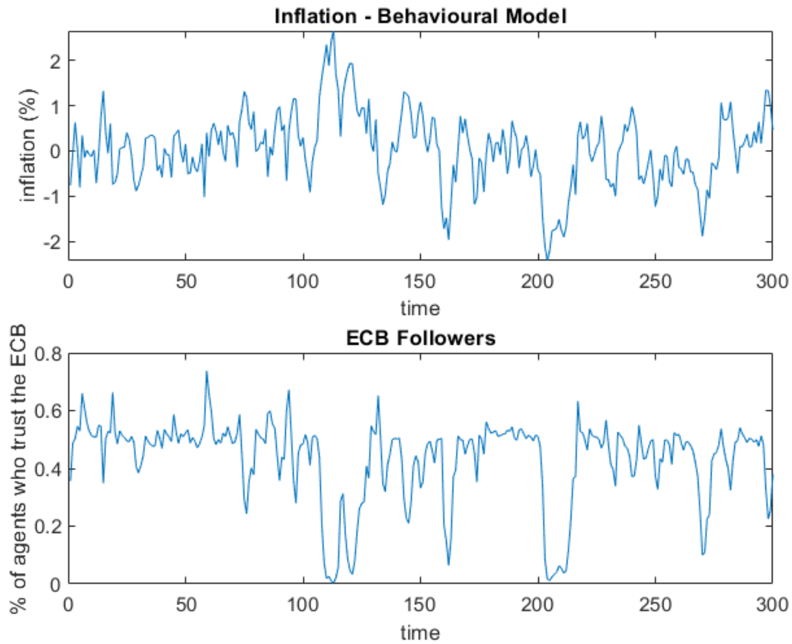

Figure 3: Inflation in the Behavioural Model
Source: produced by the author, based on De Grauwe (2012)

logue. Firstly, as illustrated in figure 4, the simulated distribution of the output gap exhibits fat tails, thereby diminishing the likelihood of underestimating extreme events, a notable shortcoming of DSGE models. Furthermore, the deviation from normality in the distribution is not attributed to the shocks, which are normally distributed, but rather arises from the endogenous dynamics of optimism and pessimism. Secondly, within this model, business cycles are endogenously determined as a result of transitions between various rules. When beliefs remain stable or fluctuate within a defined range, as observed during periods 160 to 200 in figure 2, the output gap oscillates slightly. Conversely, self-fulfilling waves of optimism and pessimism can precipitate economic booms and busts in alignment with the aforementioned process. Furthermore, it is noteworthy that the impact and persistence of shocks are significantly contingent upon the prevailing state of the market, as depicted in figure 5. The figure illustrates the impulse responses of inflation to an abrupt reduction in the interest rate r_t. In scenarios where skepticism towards the inflation target prevails, the interest rate shock exerts a significant influence on inflation. Conversely, in contexts where there is sufficient confidence in the central bank, a reduction in the interest rate exerts only a marginal impact on inflation, ”a result akin to the stabilization bonus conferred by a fully credible central bank.”

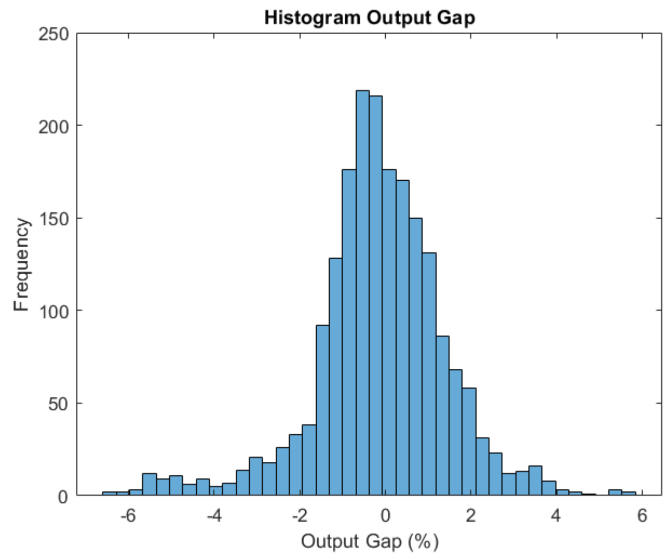

Figure 4: Frequency distribution of Output Gap in the Behavioural Model. Excess kurtosis=5.2, Jarque-Bera=489.5 (p-value=0.001)

Source: produced by the author, based on De Grauwe (2012)

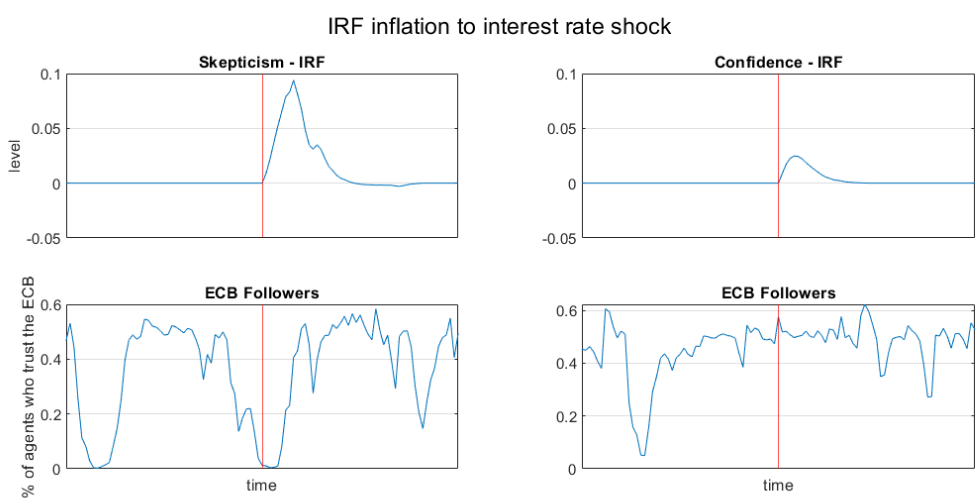

Figure 5: IRFs of inflation to a negative interest rate shock

Source: produced by the author, based on De Grauwe (2012)

Finally, the model addresses behavioural biases within an economic framework characterized by agents who possess an incomplete understanding of their environment and exhibit inefficiencies in information processing. The reliance on simple heuristics facilitates the emergence of heterogeneity, thereby abandoning the restrictive and unrealistic premise of the representative agent. Although some posit that deviating from rationality permits infinite possibilities in the formulation of ”irrational” predictions, the model demonstrates the feasibility of replicating economic dynamics when agents’ habits diverge from pure rationality, without necessitating engagement with the ”dark realm of irrationality”.

Bibliography

Abel, A. B. (1990). “Asset prices under habit formation and catching up with the Joneses”. In: *NBER Working Papers* 3279.

Brock, William and Cars Hommes (1997). “A Rational Route to Randomness”. In: *Econometrica* 65.5, pp. 1059–1096.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (2001). “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”. In: *Journal of Political Economy, University of Chicago Press* 113.1, pp. 1–45.

De Grauwe, Paul (2008). “Animal Spirits and Monetary Policy”. In: *CESifo Working Paper Series* 2418.

— (2012). *Lectures on Behavioral Macroeconomics*. Princeton University Press.

DellaVigna, Stefano (2007). “Psychology and Economics: Evidence from the Field”. In: *NBER Working Papers* 13420.

Fagan, Gabriel, Jerome Henry, and Ricardo Mestre (2001). “An area-wide model (AWM) for the euro area”. In: *Working Paper Series from European Central Bank* 42.

Galí, Jordi (2008). *Monetary Policy, Inflaction, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Galí, Jordi and Mark Gertler (2007). “Macroeconomic modeling for monetary policy evaluation”. In: *Economics Working Papers, Department of Economics and Business, Universitat Pompeu Fabra* 1039.

Goodfriend, Marvin and Robert G. King (1997). “The New Neoclassical Synthesis and the Role of Monetary Policy”. In: *NBER Macroeconomics Annual 1997* 12, pp. 231–296.

Junior, Celso Jose Costa (2016). *Understanding DSGE models*. Vernon Press Titles in Economics, Vernon Art and Science Inc.

Kahneman, Daniel and Richard H. Thaler (2006). “Anomalies: Utility Maximization and Experienced Utility”. In: *American Economic Association* 20.1, pp. 221–234.

Kahneman, Daniel and Amos Tversky (1979). “Prospect Theory: An Analysis of Decision under Risk”. In: *Econometric Society* 47.2, pp. 263–291.

Kirman, Alan (1993). "Ants, Rationality, and Recruitment". In: *The Quarterly Journal of Economics* 108.1, pp. 137–156.

Lucas, Robert E. Jr (1976). "Econometric Policy Evaluation: A Critique". In: *Carnegie-Rochester Conference Series on Public Policy, Elsevier* 1.1, pp. 19–46.

Muth, John F. (1961). "Rational Expectations and the Theory of Price Movements". In: *Econometrica* 29.3, pp. 315–335.

North, Douglass C (1993). "What Do We Mean by Rationality?" In: *Public Choice, Springer* 77.1, pp. 159–162.

Prescott, Edward C. (1986). "Theory ahead of business cycle measurement". In: *Staff Report 102, Federal Reserve Bank of Minneapolis*.

Roberts, John M (1995). "New Keynesian Economics and the Phillips Curve". In: *Journal of Money, Credit and Banking* 27.4, pp. 975–984.

Smets, Frank, Kai Christoffel, et al. (2010). "DSGE models and their use at the ECB". In: *Journal of the Spanish Economic Association* 1.1, pp. 51–65.

Smets, Frank and Rafael Wouters (2003). "An estimated dynamic stochastic general equilibrium model of the euro area". In: *Journal of the European Economic Association* 1.5, pp. 1123–1175.

— (2005). "Bayesian New Neoclassical Synthesis (NNS) Models: Modern Tools for Central Banks". In: *Journal of the European Economic Association* 3.2-3, pp. 422–433.

Taleb, Nassim Nicholas (2007). *The Black Swan: The Impact of the Highly Improbable*. Allen Lane.

Thaler, Richard (1981). "Some empirical evidence on dynamic inconsistency". In: *Economics Letters* 8.3, pp. 201–207.

Warne, Anders, Günter Coenen, and Kai Christoffel (2008). "The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis". In: *Working Paper Series, ECB* 944.

Woodford, Michael (2001). "The Taylor Rule and Optimal Monetary Policy". In: *American Economic Review, American Economic Association* 91.2, pp. 232–237.

A. Appendix

† The entirety of the subsequent code is derived from De Grauwe (2012), pages 66-67. The original code has undergone substantial modifications and expansions to accommodate a wider array of rules than those present in the original version. The Impulse Response Functions (IRF) are calculated in accordance with the methodology delineated in the same book, page 69.

1 Model

```
gam = 1;
pstar = 0;
beta = 1;
delta = 0.4;
a1 = 0.5;
a2 = -0.2;
b1 = 0.5;
b2 = 0.05;
c1 = 1.5;
c2 = 0.5;
c3 = 0.5;
A = [1 -b2;-a2*c1 1-a2*c2];
B = [b1 0;-a2 a1];
C = [1-b1 0;0 1-a1];
T = 2000;
sigma1 = 0.5;
sigma2 = 0.5;
sigma3 = 0.5;
omega=0.5;
rhoout=0.0;
rhoinf=0.0;
rhotayl=0.0;
epfs=pstar;

p = zeros(T,1);
x = zeros(T,1);
plagt = zeros(T,1);
xlagt = zeros(T,1);
r = zeros(T,1);
epf = zeros(T,1);
epc = zeros(T,1);
ep = zeros(T,1);
ex = zeros(T,1);
ERp = zeros(T,1);
FRp = zeros(T,1);
alfapt = zeros(T,1);
```

† not included in the word count

```
exfunoptt = zeros(T,1);
exfunpest = zeros(T,1);
ERx = zeros(T,1);
FORx = zeros(T,1);
FPRx = zeros(T,1);
alfaxfunoptt = zeros(T,1);
alfaxfunpest = zeros(T,1);
alfaxextt = zeros(T,1);
anspirits = zeros(T,1);
ECBfollowers = zeros(T,1);
epsilont = zeros(T,1);
etat = zeros(T,1);
ut = zeros(T,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% model %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

alfap=0.5;
alfaxfunopt=1/3;
alfaxfunpes=1/3;
alfaxext=1/3;

for t=2:T

    %definition of shocks at time t
    epsilont(t) = rhoout*epsilont(t-1) + sigma1*randn;
    etat(t)= rhoinf*etat(t-1) + sigma2*randn;
    epsilon = epsilont(t);
    eta = etat(t);
    shocks = [eta;a2*0+epsilon];

    %inflation rules
    epes=p(t-1);
    eps=alfap*epes+(1-alfap)*epfs;

    %output rules
    divt = (1-delta)*beta + delta*var(x(max(2,t-12):t)); %(1-delta)*beta=2k in the essay
    gt = divt/2;
    exext=x(t-1);
    exfunopt=gt;
    exfunpes=-gt;
    exfunoptt(t)=exfunopt;
    exfunpest(t)=exfunpes;
    exs=alfaxext*exext + alfaxfunopt*exfunopt + alfaxfunpes*exfunpes;

    %state of the model at time t
    forecast = [eps;exs];
    plag=p(t-1);
    xlag=x(t-1);
    rlag=r(t-1);
    lag = [plag;xlag];
    smooth = [0;a2*c3];
    D = B*forecast + C*lag + smooth*rlag + shocks;
    X = A\D;
    p(t)= X(1,1);
    x(t)= X(2,1);
    r(t)= c1*p(t) + c2*x(t) + c3*r(t-1)+0;
```

```
    plagt(t)=p(t-1);
    xlagt(t)=x(t-1);

    %updating rules performance and probabilities
    ERp(t) = omega*ERp(t-1) - (1-omega)*(epes-p(t))^2;
    FRp(t) = omega*FRp(t-1) - (1-omega)*(epfs-p(t))^2;
    ERx(t) = omega*ERx(t-1) - (1-omega)*(exext-x(t))^2;
    FORx(t) = omega*FORx(t-1) - (1-omega)*(exfunopt-x(t))^2;
    FPRx(t) = omega*FPRx(t-1) - (1-omega)*(exfunpes-x(t))^2;
    alfap = exp(gam * ERp(t)) / (exp(gam * ERp(t)) + exp(gam * FRp(t)));
    alfaxext = exp(gam * ERx(t)) / (exp(gam * ERx(t)) + exp(gam * FORx(t)) + exp(gam * FPRx(t)));
    alfaxfunopt = exp(gam * FORx(t)) / (exp(gam * ERx(t)) + exp(gam * FORx(t)) + exp(gam * FPRx(t)));
    alfaxfunpes = exp(gam * FPRx(t)) / (exp(gam * ERx(t)) + exp(gam * FORx(t)) + exp(gam * FPRx(t)));
    alfapt(t) = alfap;
    alfaxextt(t) = alfaxext;
    alfaxfunoptt(t) = alfaxfunopt;
    alfaxfunpest(t) = alfaxfunpes;

    if exext>0
        anspirits(t)=alfaxext + alfaxfunopt;
    end
    if exext<=0
        anspirits(t)=alfaxfunopt;
    end

    ECBfollowers(t)=1-alfap;
end

%kurtosis & jbtest
Kurt = kurtosis(x);
[jb,pvalue,jbstat] = jbtest(x,0.05);
disp(['4th Moment (Kurtosis): ', num2str(Kurt)]);
disp('Jarque-Bera Test Results:');
disp(['JB Statistic: ', num2str(jbstat)]);
disp(['p-value: ', num2str(pvalue)]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% charts %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure;
histogram(x, 40)
title('Histogram Output Gap');
xlabel('Output Gap (%)');
ylabel('Frequency');

figure;
subplot(2,1,1)
plot(x(301:600))
title('Output Gap - Behavioural Model')
xlabel('time')
ylabel('output gap (%)')
xlim([0 300])
subplot(2,1,2)
plot(anspirits(301:600))
title('Animal Spirits')
xlabel('time')
ylabel('% of agents with positive forecast')
```

```
xlim([0 300])

figure;
subplot(2,1,1)
plot(p(301:600))
title('Inflation - Behavioural Model')
xlabel('time')
ylabel('inflation (%)')
subplot(2,1,2)
plot(ECBfollowers(301:600))
title('ECB Followers')
xlabel('time')
ylabel('% of agents who trust the ECB')
xlim([0 300])
```

2 IRF

```
gam = 1;
pstar = 0;
beta = 1;
delta = 0.4;
a1 = 0.5;
a2 = -0.2;
b1 = 0.5;
b2 = 0.05;
c1 = 1.5;
c2 = 0.5;
c3 = 0.5;
A = [1 -b2;-a2*c1 1-a2*c2];
B = [b1 0;-a2 a1];
C = [1-b1 0;0 1-a1];
T = 1000;
sigma1 = 0.5;
sigma2 = 0.5;
sigma3 = 0.5;
omega=0.5;
rhoout=0.0;
rhoinf=0.0;
rhotayl=0.0;
epfs=pstar;
epsilont = zeros(T,1);
etat = zeros(T,1);
ut = zeros(T,1);

Baseline_p = zeros(T,1);
Baseline_x = zeros(T,1);
Baseline_plagt = zeros(T,1);
Baseline_xlagt = zeros(T,1);
Baseline_r = zeros(T,1);
Baseline_epf = zeros(T,1);
Baseline_epc = zeros(T,1);
Baseline_ep = zeros(T,1);
Baseline_ex = zeros(T,1);
Baseline_ERp = zeros(T,1);
Baseline_FRp = zeros(T,1);
```

```
Baseline_alfapt = zeros(T,1);
Baseline_exfunoptt = zeros(T,1);
Baseline_exfunpest = zeros(T,1);
Baseline_ERx = zeros(T,1);
Baseline_FORx = zeros(T,1);
Baseline_FPRx = zeros(T,1);
Baseline_alfaxfunoptt = zeros(T,1);
Baseline_alfaxfunpest = zeros(T,1);
Baseline_alfaxextt = zeros(T,1);
Baseline_anspirits = zeros(T,1);
Baseline_ECBfollowers = zeros(T,1);

Perturbed_p = zeros(T,1);
Perturbed_x = zeros(T,1);
Perturbed_plagt = zeros(T,1);
Perturbed_xlagt = zeros(T,1);
Perturbed_r = zeros(T,1);
Perturbed_epf = zeros(T,1);
Perturbed_epc = zeros(T,1);
Perturbed_ep = zeros(T,1);
Perturbed_ex = zeros(T,1);
Perturbed_ERp = zeros(T,1);
Perturbed_FRp = zeros(T,1);
Perturbed_alfapt = zeros(T,1);
Perturbed_exfunoptt = zeros(T,1);
Perturbed_exfunpest = zeros(T,1);
Perturbed_ERx = zeros(T,1);
Perturbed_FORx = zeros(T,1);
Perturbed_FPRx = zeros(T,1);
Perturbed_alfaxfunoptt = zeros(T,1);
Perturbed_alfaxfunpest = zeros(T,1);
Perturbed_alfaxextt = zeros(T,1);
Perturbed_anspirits = zeros(T,1);
Perturbed_ECBfollowers = zeros(T,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% model %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

u=0.0;
control=0;

Baseline_alfap=0.5;
Baseline_alfaxfunopt=1/3;
Baseline_alfaxfunpes=1/3;
Baseline_alfaxext=1/3;

Perturbed_alfap=0.5;
Perturbed_alfaxfunopt=1/3;
Perturbed_alfaxfunpes=1/3;
Perturbed_alfaxext=1/3;

for t=2:T

    %definition of shocks at time t
    epsilont(t) = rhoout*epsilont(t-1) + sigma1*randn;
    etat(t)= rhoinf*etat(t-1) + sigma2*randn;
```

```
l = find(Baseline_ECBfollowers, 1, 'last');
%change second argument to get left panel
if t>100 && Baseline_ECBfollowers(l) >= 0.55 && control == 0
    ut(t) = -1;
    u = ut(t);
    control = 1;
    bar = l;
else
    ut(t) = 0.6*ut(t-1);
    u = ut(t);
end

epsilon = epsilont(t);
eta = etat(t);

Baseline_shocks = [eta;epsilon];
Perturbed_shocks = [eta;a2*u+epsilon];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% baseline model %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%inflation rules
Baseline_epes=Baseline_p(t-1);
Baseline_eps=Baseline_alfap*Baseline_epes+(1-Baseline_alfap)*epfs;

%output rules
Baseline_divt = (1-delta)*beta+delta*var(Baseline_x(max(2,t-12):t));
Baseline_gt = Baseline_divt/2;
Baseline_exext=Baseline_x(t-1);
Baseline_exfunopt=Baseline_gt;
Baseline_exfunpes=-Baseline_gt;
Baseline_exfunoptt(t)=Baseline_exfunopt;
Baseline_exfunpest(t)=Baseline_exfunpes;
Baseline_exs=Baseline_alfaxext*Baseline_exext+Baseline_alfaxfunopt*Baseline_exfunopt+Baseline_alfaxfunpes*
     Baseline_exfunpes;

%state of the model at time t
Baseline_forecast = [Baseline_eps;Baseline_exs];
Baseline_plag=Baseline_p(t-1);
Baseline_xlag=Baseline_x(t-1);
Baseline_rlag=Baseline_r(t-1);
Baseline_lag = [Baseline_plag;Baseline_xlag];
Baseline_smooth = [0;a2*c3];
Baseline_D = B*Baseline_forecast + C*Baseline_lag + Baseline_smooth*Baseline_rlag + Baseline_shocks;
Baseline_X = A\Baseline_D;
Baseline_p(t)= Baseline_X(1,1);
Baseline_x(t)= Baseline_X(2,1);
Baseline_r(t)= c1*Baseline_p(t)+c2*Baseline_x(t)+c3*Baseline_r(t-1)+u;
Baseline_plagt(t)=Baseline_p(t-1);
Baseline_xlagt(t)=Baseline_x(t-1);

%updating rules performance and probabilities
Baseline_ERp(t) = omega*Baseline_ERp(t-1) - (1-omega)*(Baseline_epes-Baseline_p(t))^2;
Baseline_FRp(t) = omega*Baseline_FRp(t-1) - (1-omega)*(epfs-Baseline_p(t))^2;
Baseline_ERx(t) = omega*Baseline_ERx(t-1) - (1-omega)*(Baseline_exext-Baseline_x(t))^2;
Baseline_FORx(t) = omega*Baseline_FORx(t-1) - (1-omega)*(Baseline_exfunopt-Baseline_x(t))^2;
Baseline_FPRx(t) = omega*Baseline_FPRx(t-1) - (1-omega)*(Baseline_exfunpes-Baseline_x(t))^2;
```

```
Baseline_alfap = exp(gam*Baseline_ERp(t))/(exp(gam * Baseline_ERp(t)) + exp(gam * Baseline_FRp(t)));
Baseline_alfaxext = exp(gam*Baseline_ERx(t))/(exp(gam * Baseline_ERx(t)) + exp(gam * Baseline_FORx(t)) + exp(gam *
     Baseline_FPRx(t)));
Baseline_alfaxfunopt = exp(gam*Baseline_FORx(t))/(exp(gam * Baseline_ERx(t)) + exp(gam * Baseline_FORx(t)) + exp(gam *
      Baseline_FPRx(t)));
Baseline_alfaxfunpes = exp(gam*Baseline_FPRx(t))/(exp(gam * Baseline_ERx(t)) + exp(gam * Baseline_FORx(t)) + exp(gam *
      Baseline_FPRx(t)));
Baseline_alfapt(t) = Baseline_alfap;
Baseline_alfaxextt(t) = Baseline_alfaxext;
Baseline_alfaxfunoptt(t) = Baseline_alfaxfunopt;
Baseline_alfaxfunpest(t) = Baseline_alfaxfunpes;

if Baseline_exext>0
    Baseline_anspirits(t)=Baseline_alfaxext + Baseline_alfaxfunopt;
end
if Baseline_exext<=0
    Baseline_anspirits(t)=Baseline_alfaxfunopt;
end

Baseline_ECBfollowers(t)=1-Baseline_alfap;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% perturbed model %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%inflation rules
Perturbed_epes=Perturbed_p(t-1);
Perturbed_eps=Perturbed_alfap*Perturbed_epes+(1-Perturbed_alfap)*epfs;

%output rules
Perturbed_divt = (1-delta)*beta+delta*var(Perturbed_x(max(2,t-12):t));
Perturbed_gt = Perturbed_divt/2;
Perturbed_exext=Perturbed_x(t-1);
Perturbed_exfunopt=Perturbed_gt;
Perturbed_exfunpes=-Perturbed_gt;
Perturbed_exfunoptt(t)=Perturbed_exfunopt;
Perturbed_exfunpest(t)=Perturbed_exfunpes;
Perturbed_exs=Perturbed_alfaxext*Perturbed_exext+Perturbed_alfaxfunopt*Perturbed_exfunopt+Perturbed_alfaxfunpes*
     Perturbed_exfunpes;

%state of the model at time t
Perturbed_forecast = [Perturbed_eps;Perturbed_exs];
Perturbed_plag=Perturbed_p(t-1);
Perturbed_xlag=Perturbed_x(t-1);
Perturbed_rlag=Perturbed_r(t-1);
Perturbed_lag = [Perturbed_plag;Perturbed_xlag];
Perturbed_smooth = [0;a2*c3];
Perturbed_D = B*Perturbed_forecast + C*Perturbed_lag + Perturbed_smooth*Perturbed_rlag + Perturbed_shocks;
Perturbed_X = A\Perturbed_D;
Perturbed_p(t)= Perturbed_X(1,1);
Perturbed_x(t)= Perturbed_X(2,1);
Perturbed_r(t)= c1*Perturbed_p(t)+c2*Perturbed_x(t)+c3*Perturbed_r(t-1)+u;
Perturbed_plagt(t)=Perturbed_p(t-1);
Perturbed_xlagt(t)=Perturbed_x(t-1);

%updating rules performance and probabilities
Perturbed_ERp(t) = omega*Perturbed_ERp(t-1) - (1-omega)*(Perturbed_epes-Perturbed_p(t))^2;
```

```
    Perturbed_FRp(t) = omega*Perturbed_FRp(t-1) - (1-omega)*(epfs-Perturbed_p(t))^2;
    Perturbed_ERx(t) = omega*Perturbed_ERx(t-1) - (1-omega)*(Perturbed_exext-Perturbed_x(t))^2;
    Perturbed_FORx(t) = omega*Perturbed_FORx(t-1) - (1-omega)*(Perturbed_exfunopt-Perturbed_x(t))^2;
    Perturbed_FPRx(t) = omega*Perturbed_FPRx(t-1) - (1-omega)*(Perturbed_exfunpes-Perturbed_x(t))^2;
    Perturbed_alfap = exp(gam*Perturbed_ERp(t))/(exp(gam * Perturbed_ERp(t)) + exp(gam * Perturbed_FRp(t)));
    Perturbed_alfaxext = exp(gam*Perturbed_ERx(t))/(exp(gam * Perturbed_ERx(t)) + exp(gam * Perturbed_FORx(t)) + exp(gam *
         Perturbed_FPRx(t)));
    Perturbed_alfaxfunopt = exp(gam*Perturbed_FORx(t))/(exp(gam * Perturbed_ERx(t)) + exp(gam * Perturbed_FORx(t)) + exp(
        gam * Perturbed_FPRx(t)));
    Perturbed_alfaxfunpes = exp(gam*Perturbed_FPRx(t))/(exp(gam * Perturbed_ERx(t)) + exp(gam * Perturbed_FORx(t)) + exp(
        gam * Perturbed_FPRx(t)));
    Perturbed_alfapt(t) = Perturbed_alfap;
    Perturbed_alfaxextt(t) = Perturbed_alfaxext;
    Perturbed_alfaxfunoptt(t) = Perturbed_alfaxfunopt;
    Perturbed_alfaxfunpest(t) = Perturbed_alfaxfunpes;

    if Perturbed_exext>0
        Perturbed_anspirits(t)=Perturbed_alfaxext + Perturbed_alfaxfunopt;
    end
    if Perturbed_exext<=0
        Perturbed_anspirits(t)=Perturbed_alfaxfunopt;
    end

    Perturbed_ECBfollowers(t)=1-Perturbed_alfap;
end

p = Perturbed_p - Baseline_p;
x = Perturbed_x - Baseline_x;

%the script must be run twice: first for the left panel (assigning p1=p;bar1=bar), then for the right panel.
% figure('Name','IRF inflation to interest rate shock');
% sgtitle('IRF inflation to interest rate shock')
% subplot(2,2,1)
% plot(p1)
% title('Skepticism - IRF')
% ylabel('level')
% xline(bar1, 'r')
% xlim([bar1-50 bar1+50])
% ylim([-.05 .1])
% xticks([])
% grid on;
% subplot(2,2,3)
% plot(Baseline_ECBfollowers1)
% title('ECB Followers')
% xlabel('time')
% ylabel('% of agents who trust the ECB')
% xline(bar1, 'r')
% xlim([bar1-50 bar1+50])
% xticks([])
% grid on;
% subplot(2,2,2)
% plot(p)
% title('Confidence - IRF')
% xline(bar, 'r')
% xlim([bar-50 bar+50])
% ylim([-.05 .1])
% xticks([])
% grid on;
```

```
% subplot(2,2,4)
% plot(Baseline_ECBfollowers)
% title('ECB Followers')
% xlabel('time')
% xline(bar, 'r')
% xlim([bar-50 bar+50])
% xticks([])
% grid on;
```