Forward Guidance

The 3-Equation NK System

Consider the linearized New Keynesian model:

$$x_t = -(i_t - \tilde{E}_t \pi_{t+1} - r_t^*) + \tilde{E}_t x_{t+1} \tag{1}$$
$$\pi_t = \kappa x_t + \beta \tilde{E}_t \pi_{t+1} + u_t \tag{2}$$
$$i_t = \phi_\pi \pi_t \tag{3}$$

where x_t is the output gap, π_t is inflation, i_t is the nominal interest rate, r_t^* is the efficient real interest rate, u_t is a cost-push shock, $\kappa > 0$, $0 < \beta < 1$, and $\phi_\pi > 0$.

Forward Guidance in the Baseline Model

Assume first that expectations are rational, so that $\tilde{E}_t = E_t$.

(a) Starting from

$$x_t = -(i_t - E_t \pi_{t+1} - r_t^*) + E_t x_{t+1}, \tag{4}$$

iterate the equation forward and show that

$$x_t = -E_t \sum_{j=0}^{\infty} \left(i_{t+j} - \pi_{t+j+1} - r_{t+j}^* \right). \tag{5}$$

(b) Explain why, in this formulation, an announced future fall in the nominal interest rate affects current output even if the fall is arbitrarily far in the future.

(c) Consider a forward-guidance experiment in which the central bank announces at time t that the nominal interest rate will be lower than previously expected only at time $t + T$, for some large T. Using the expression above, explain why the effect on x_t does not vanish as T becomes large.

(d) Explain why this feature is called the *forward guidance puzzle.*

Discounted Aggregate Demand and the Puzzle

Suppose now that aggregate demand instead takes the form

$$x_t = -(i_t - E_t \pi_{t+1} - r_t^*) + \delta E_t x_{t+1}, \qquad 0 < \delta < 1. \tag{6}$$

(a) Compute the forward recursion and show that

$$x_t = -E_t \sum_{j=0}^{\infty} \delta^j \left(i_{t+j} - \pi_{t+j+1} - r_{t+j}^* \right). \tag{7}$$

(b) Compare this expression with the baseline case. Which term is responsible for damping the effect of distant future policy announcements?

(c) Explain why introducing discounting in aggregate demand can mitigate the forward guidance puzzle.

Stubborn Agents

Now suppose that a fraction $\gamma \in (0,1)$ of agents have rational expectations, while the remaining fraction $1-\gamma$ always expect all endogenous variables next period to revert to steady state. The exam question states precisely this environment.

(a) Show that under this assumption,

$$\tilde{E}_t x_{t+1} = \gamma E_t x_{t+1}, \qquad \tilde{E}_t \pi_{t+1} = \gamma E_t \pi_{t+1}. \tag{8}$$

(b) Rewrite aggregate demand and aggregate supply in terms of the rational expectations operator $E_t(\cdot)$ only.

(c) Explain why, relative to the fully rational benchmark, the effect of future expected variables is dampened.

(d) Explain why this mechanism can potentially solve the forward guidance puzzle.

Matrix Representation with Stubborn Agents

Define

$$y_t \equiv \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}, \qquad v_t \equiv \begin{bmatrix} r_t^* \\ u_t \end{bmatrix}. \tag{9}$$

(a) Starting from the rewritten system, show that the equilibrium can be written in the form

$$E_t y_{t+1} = A y_t + B v_t. \tag{10}$$

(b) Derive the matrix A explicitly as a function of $\beta, \kappa, \phi_\pi, \gamma$.

(c) Verify that when $\gamma = 1$ your matrix A collapses to the standard rational-expectations case.

(d) Explain informally what changes in the dynamics when γ falls.

Determinacy with Stubborn Agents

(a) Derive the conditions under which the equilibrium is unique and determinate.

(b) Explain why reducing γ tends to make determinacy easier to obtain.

(c) Consider the case of an interest-rate peg:

$$\phi_\pi = 0. \tag{11}$$

Can equilibrium still be determinate? Show carefully which condition is relevant and explain the intuition.

Cognitive Discounting

Now agents know the steady state exactly, but they down-weight the expected log-deviation of a variable from steady state k periods ahead by a factor m^k, where $0 < m < 1$.

(a) Show that, for one-step-ahead expectations, cognitive discounting implies

$$\tilde{E}_t x_{t+1} = m E_t x_{t+1}, \qquad \tilde{E}_t \pi_{t+1} = m E_t \pi_{t+1}. \tag{12}$$

(b) Rewrite the NK system in terms of the usual rational-expectations operator $E_t(\cdot)$.

(c) Explain why this specification is conceptually similar to replacing the forward-looking term in aggregate demand with a discounted continuation value.

(d) Explain why cognitive discounting can potentially solve the forward guidance puzzle.

Matrix Representation with Cognitive Discounting

Using the same notation

$$y_t \equiv \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}, \qquad v_t \equiv \begin{bmatrix} r_t^* \\ u_t \end{bmatrix}, \tag{13}$$

consider the equilibrium representation

$$\tilde{E}_t y_{t+1} = A y_t + B v_t. \tag{14}$$

(a) Derive the matrix A explicitly as a function of $\beta, \kappa, \phi_\pi, m$.

(b) Compare this matrix with the one obtained under stubborn agents. Which parameter plays the analogous role?

(c) Verify that when $m = 1$ you recover the fully rational benchmark.

Determinacy with Cognitive Discounting

(a) Derive the conditions under which equilibrium is unique and determinate.

(b) Explain why cognitive discounting tends to shrink the effect of distant future policy announcements.

(c) Consider again the interest-rate peg:

$$\phi_\pi = 0. \tag{15}$$

Can the equilibrium still be determinate? State the condition and explain the intuition.